# Memory Based_JEE Main Online Test_07-01-20_Morning

## Physics

**1.** A hydrogen has electron in ground state with time period T = 1.6 × $10^{-16}$ s then find the frequency of revolution in n = 2 state.

**Sol.** $T \propto \frac{n^3}{z^2}$

$$\frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^3 = \left(\frac{2}{1}\right)^3 = 8$$

$\Rightarrow$ $T_2 = 12.8 \times 10^{-16}$ s

$\therefore$ $f = \frac{1}{T_2} = \frac{100}{12.8} \times 10^{14}$

$f = 7.81 \times 10^{14}$ Hz

**2.** In a single slit diffraction, if 2nd minima is at 60° then the Ist minima will exit at.

**Sol.** for second minima :

$a \sin\theta_2 = 2\lambda$

for Ist minima

$a.\sin\theta_1 = \lambda$

$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$

$$\sin\theta_1 = \frac{1}{2}\sin 60^\circ = \frac{\sqrt{3}}{4}$$

$$\theta_1 = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

**3.** Two gas A and B are mixed together, gas-A contains 2-mol and gas B contains 3-mol,

Given $\gamma_A = \frac{4}{3}$

$\gamma_B = \frac{5}{3}$

find $\gamma_{mixture}$ = ...

**Sol.** From $\gamma = 1 + \frac{2}{f}$ ...(i)

$f_A = 6$,

$f_B = 3$,

$$f_{mixture} = \frac{2 \times 6 + 3 \times 3}{5} = \frac{21}{5}$$

Hence, from (i) $\gamma_{mixture} = \frac{31}{21}$

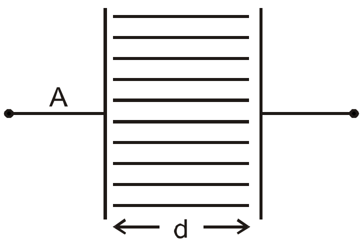
**4.** Magnetic field : $\vec{B} = 3\times10^{-8}\sin\left(5\times10^{8}t+10^{-9}x\right)\hat{j}$

find the Electric field of respective EM- wave.

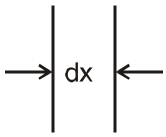
**Sol.** From C = $\frac{E}{B}$ and $\hat{E} = \left(\hat{C}\times\hat{B}\right)$ = $-\hat{i}\times\hat{j} = -\hat{k}$

$\vec{E} = 9\sin\left(5\times10^{8}t+10^{-9}x\right)\left(-\hat{k}\right)$

**5.** For given parallel plate capacitor if dielectric constant varies with distance as k(1 + $\alpha$x). Find the equivalent capacitance.




**Sol.** Considering dx section at x.



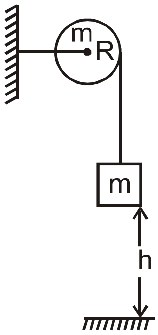

dc = $\frac{\varepsilon_0 k(1+\alpha x)A}{dx}$

$\therefore \quad \int d\left(\frac{1}{C_{eq}}\right) = \int_0^d \frac{dx}{k\varepsilon_0 A(1+\alpha x)}$

$\Rightarrow \quad \left(\frac{1}{C_{eq}}\right) = \frac{1}{k\varepsilon_0 A}\ell n(1+\alpha d)$

$\therefore \quad C_{eq} = \frac{k\varepsilon_0 A}{\ell n(1+\alpha d)}$

**6.** Find angular speed of the pulley after mass M drops by a distance h. Assume pulley is a disc of mass m and radius R.




**Sol.** From conservation of energy

$$\Delta KE = W_{\text{all forces}} \Rightarrow \frac{1}{2}\frac{mR^2}{2}\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{mR^2\omega^2}{4} + \frac{m\omega^2R^2}{2} = mgh$$

$$\frac{3mR^2\omega^2}{4} = mgh$$

$$\omega = \sqrt{\frac{4gh}{3R^2}}$$

**7.** LCR oscillation is compared with the spring mass system dampes oscillation. (b $\rightarrow$ Damamping constant). Value of L, C and R in comparison to damped oscillation ?

**Sol.** $L \rightarrow M$

$R \rightarrow b$

$$C \rightarrow \frac{1}{K}$$

**8.** 1L gas at STP, expands adiabatically to 3L.Find the work done ($\gamma$ = 1.4)

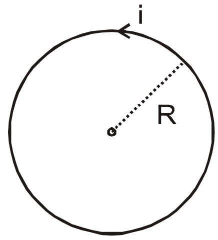
**Sol.** $W = \dfrac{P_2V_2 - P_1V_1}{1-\gamma}$

$P_1 (1)^\gamma = P_2 (3)^\gamma$

$P_2 = 0.214$

$$w = \frac{0.644 - 1}{1 - 1.4} = 0.89$$

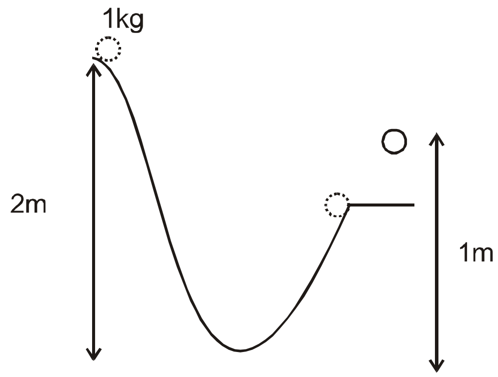
**9.** For a constant carrying will carying current the magnetic flux passing through the coil is $\phi_{in}$ and flux passing outside the coil is $\phi_{in}$ then,




(1) $\phi_{in} \neq \phi_{out}$ (2) $\phi_{in} = \phi_{out}$ (3) $\phi_{in} = \frac{1}{2}\phi_{out}$ (4) $\phi_{in} = 2\phi_{out}$

**Sol.** **(2)**
Magnetic field lines are formed in closed loops hence no. of field lines passing inside the loops are equal to number of fied lines passing through outside of the loop.

**10.** Find the kinetic energy at point P, at the height of 1m :




**Sol.** Total work done = $\Delta$KE
$\Rightarrow (1)(2)(10) - (1)(1)(10)$
F.K.E – I.K.E $\Rightarrow$ 20 – 10
F.K.E – 0 = 10
K.E. = 10 J

**11.** A Beam of electromagnetic radiation of intensity $64 \times 10^{-5}$ w/cm$^2$ is compaised of wavelength $\lambda$ = 310 It falls normally on a metal (work function $\phi$ = 2ev) of surface area 1 cm$^2$. If one in $10^3$ photons ejects an electron, total number of electrons ejected in is $10^x$ (hc = 1240 ev nm ; 1ev = $1.6 \times 10^{-19}$J) then x is

**Sol.** $I = h\left(\frac{E}{At}\right)$

$\because \quad E = \frac{hc}{\lambda} = \frac{1240}{310}$

E = 4ev

$\frac{n}{t} = \frac{IA}{E}$

$= \frac{6.4 \times 10^{-5} \times 1}{4 \times 1.6 \times 10^{-19}}$

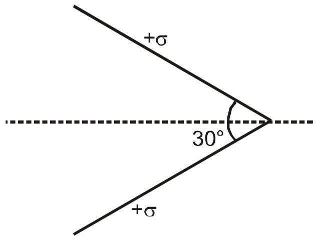
$\frac{n}{t} = 10^{14}$ photon/sec

to electron ejected = $\frac{10^{14}}{10^{3}} = 10^{11}$

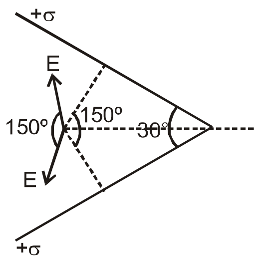
on comparing $10^{x}$ to $10^{11}$

$x = 11$

**12.** Two infinite sheet are inclined to each other at 30° & carry a positive surface charge density $\sigma$ then find electric field in the region between them .

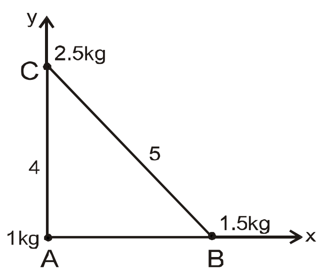



**Sol.**




$\frac{\sigma}{2\in_0}\left[-\frac{1}{2}\hat{i}+\left(1-\frac{\sqrt{3}}{2}\right)\hat{i}\right]$ ; $E_r = \sqrt{E^2+E^2+2E^2.\cos(150^\circ)}$ ; $E_r = \frac{\sigma}{2\in_0}\sqrt{2-\sqrt{3}}$

**13.** Find center of mass of given mass distribution.




**Sol.** $X_{com} = \frac{1\times 0+1.5\times 3+2.5\times 0}{5} = 0.9$

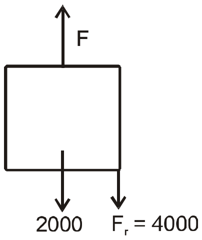
$y_{com} = \frac{1\times 0+1.5\times 0+2.5\times 4}{5} = 2$

(0.9, 2)

0.9 m right of A and 2 m above A.

**14.** Maximum load lifted by a motor delivering power of 60 hp is 2000N. Friction force of 4000N is also acting opposite to the motion of the lift . Find velocity with which the load is being raised ? (1 hp = 746 watts)

**Sol.**

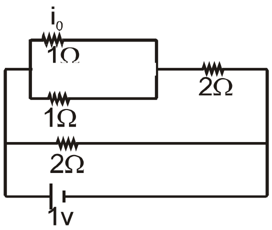



F = 6000

P = FV

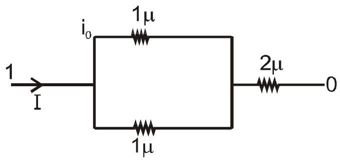
7.46 m/s

**15.** Find $i_0$ ?



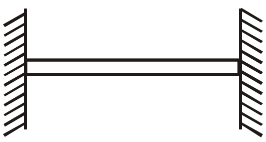

**Sol.** $I = \frac{1}{2.5} = \frac{2}{5}$

$i_0 = \frac{I}{2} = \frac{1}{5}A$




**16.** Speed of transverse wave of a straight wire (Mass 2 kg, length 20 cm and area of corss section 20 mm$^2$) is 90 m/s. If the young modulus of wire is 1.6 × 10$^{11}$ N/m$^2$. The extention of the wire over its natural is.

**Sol.**



$\mu = \frac{m}{\ell} = \frac{2}{0.2} = 10$

$V = \sqrt{\frac{T}{\mu}} = 90 \Rightarrow \frac{T}{10} = 8100 \Rightarrow T = 81000$

$y = \frac{T/A}{\Delta\ell/\ell} \Rightarrow \Delta\ell = \frac{T\ell}{Ay}$

$\Delta\ell = \frac{81000 \times 0.2}{0.2 \times 10^{-4} \times 1.6 \times 10^{11}} = 50.625 \times 10^{-4}$

**17.** A polarizer-analyzer set is adjusted the intensity of light coming out of the analyzer is just 10% of the original intensity. Assuming that polarizer-analyzer set does not absorved any light the anlge by which analyzer needs to be rotated further to reduce the output intensity to be zero is.

**Sol.** Intensity 10% intensity is transmitted.

$I = I_0 \cos^2\theta$

$0.1\, I_0 = I_0 \cos^2\theta\, 1. \quad \Rightarrow \quad \sec^2\theta = 10$

$\tan\theta = 3 \qquad \theta = 71.6$

Now further angle of rotation = 90 – 71.6

to get zero intensity = 18.4.

**18.** A carrot engine operates between two reservoir of temperature 900K and 300K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the law temperature reservoir, in a cycle is:

**Sol.** $W = Q_L + Q_H = 1200$

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} = \frac{300}{900} = \frac{1}{3}$$

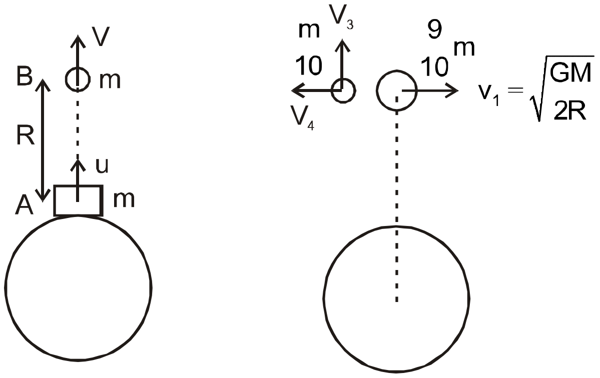
$Q_H = 3Q_L$

$4Q_L = 1200$

$Q_L = 300$ J

**19.** A satllite of mass m is launched vertically upwards with an initial speed u from surface of earth after it reaches height R ( R = Radius of the earth). it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is ( G is the gravitational constant ; m is the mass of the earth)

**Sol.**




Energy conservation from A to B

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$mv^2 = mv^2 - \frac{GMm}{R}$$

At rocket firing:-

Using conservation of momentum.

$mv = \frac{m}{10}v_3$ $\Rightarrow v_3 = 10V$

$$\frac{m}{10}v_4 = \frac{9}{10}mv_1 \Rightarrow v_4 = 9v_1 = 9\sqrt{\frac{GM}{2R}}$$

K.E of Rocket $K = \frac{1}{2}\left(\frac{M}{10}\right)\left(v_3^2 + v_4^2\right)$

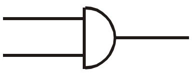
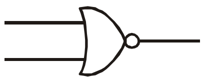
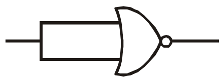
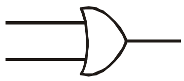
$$k = \frac{1}{2}\left(\frac{M}{10}\right)\left(100^2 + \frac{81}{2}\frac{GM}{R}\right)$$

$$k = \frac{1}{2}\left(\frac{m}{10}\right)\left(100\left(v^2 - \frac{GM}{R}\right) + \frac{81}{2}\frac{GM}{R}\right)$$

$$k = 5m\left(v^2 \frac{GM}{R} + \frac{81GM}{200R}\right)$$

$$k = 5m\left(v^2 - \frac{119GM}{200R}\right)$$

**20.** Which of the following gives a reversible operation?

(1)  (2)  (3*)  (4) 

**Ans.** **(3)**

**21.** A long solenoid of radius R carries a time (t) dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near to middle the induced EMF ($V_R$) in the ring change as:

**Sol.** Induced EMF = $E = -\frac{d\phi}{dt} = \frac{-d}{dt}\left(\mu_0 ni\left(\pi R^2\right)\right)$

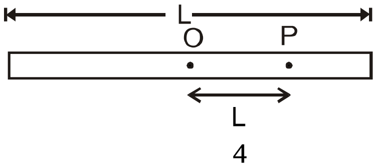
$$E = -\mu_0 n\pi R^2 \frac{d}{dt}(i)$$

$$E = -\mu_0 n\pi R^2 \frac{d}{dt}\left(I_0\left(t - t^2\right)\right)$$

$$E = -\mu_0 n\pi R^2\left(I_0\left(1 - 2t\right)\right)$$

**22.** The radius of gyration of a uniform rod of length L, about an axis passing through a ponit $\frac{L}{4}$ away from the centre of the rod and perpendicular to it is.

**Sol.**




$$I_P = I_{cm} + m\left(\frac{L}{4}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{16}$$

$$I_p = \frac{7mL^2}{48}$$

Radius of gyration $K = \sqrt{\frac{I}{m}} = \left(\sqrt{\frac{7}{48}}\right)L$

**23.** If we need a magnification of 375 from a compound microscope of tube length 150 mm and or objective of focal length 5 mm the focal length of the eye-piece should be

**Sol.** If the final image is found at infinity then

$$m = \frac{L}{f_0} \times \frac{D}{f_e}$$

$L \rightarrow$ length of tube

$f_0 \rightarrow$ focal length of objective
$f_e \rightarrow$ focal length of eye piece.
$D \rightarrow$ Distinct vision = 25 cm = 250 mm

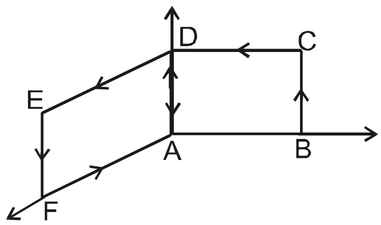
$$375 = \frac{150 \times 250}{5 \times f_e}$$

$$f_e = \frac{150 \times 50}{5 \times 375} = 20 \text{ mm}$$

**24.** A loop ABCDEFA of straight edges has six corner point A (0, 0, 0); B(5,0,0) ;C(5, 5, 0) ; D(0, 5, 0) E(0, 5, 5) and F(0, 0, 5) . The M.F. in this region is $\vec{B} = (3i + 4\hat{k})$T. The quantity of flux through the loop ABCDEFA is

**Sol.** $\phi = \vec{B}.\vec{A}$

$\phi = \phi_{ABCD} + \phi_{ADEF}$




$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\phi = (3i + 4k).(25\hat{k}) + (3\hat{i} + 4\hat{k}).25\hat{i}$

$= 100 + 75 = 175$Wb

**25.** For a given material coefficient of linear expansion along x axis is $\alpha_1$ and along y and z axes is $\alpha_2$ then if the coefficient of volume expansion is C × $10^{-6}$ / °C. Then find C.

$\alpha_1 = 5 \times 10^{-5}$ / °C $\qquad \alpha_y = \alpha_z = 5 \times 10^{-6}$ / °C

**Sol.** $\gamma = \alpha_1 + 2\alpha_2$

$\gamma = \alpha_x + \alpha_y + \alpha_z$

$\gamma = \alpha_x + 2\alpha_y$

$\gamma = 50 \times 10^{-6} + 2(5 \times 10^{-6})$

$\gamma = 60 \times 10^{-6}$ / °C

C = 60

# Memory Based_JEE Main Online Test_07-01-20_Morning

## Chemistry

**1.** Which element has the closest radius to Ag ?

(1) Cu (*2) Au (3) Hg (4) ............

**Ans.** **Au 2.** Consider the theory that can explain the com-

plex $Ni(CO)_4$ is

(*1) Valence bond theory (2) molecular orbital theory (3) crystal field theory (4) Wernor's theory

**Ans.** **Valence bond theory**

**3.** Compounds of potassium $K_2O$, $KO_2$, $K_2O_2$ the oxidation state of " K " in these respectively are :

**Ans.** **+1 , +1 , +1**

**4.** The correct order of dipole moment for $CCl_4$, $CHCl_3$, $CH_4$ is

**Ans.** $\mathbf{CHCl_3 > CCl_4 = CH_4}$

**5.** The IUPAC nomenclature of the following complex is

$[Pt(NH_3)_2(CH_3NH_2)Cl]Cl$

**Ans.** **diamminechloridomethylamine platinum (II) chloride**

**6.** The purest form of commercial iron is :

**Ans.** **Wrought iron**

**7.** $\Delta H_{eg}$ of F, Cl , Br, and I is respectively

**Ans.** **–328, –349, –325, –296**

**8.** The correct order for molecular forces of attraction is :

**Ans.** **ion - ion > ion -dipole > dipole - dipole**

**9.** The reaction

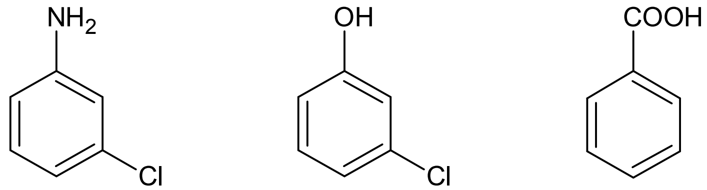
$$Cl_2 + NaOH\ (\text{hot \& Conc.}) \longrightarrow X + Y$$

$$X + Y \xrightarrow{AgNO_3} (\text{White ppt})$$

The bond order of Cl – O in oxyanion of chlorine

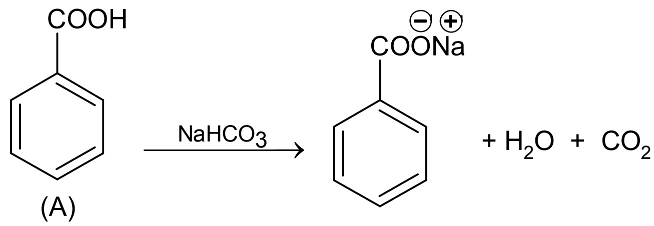
**Ans.** **1.67**

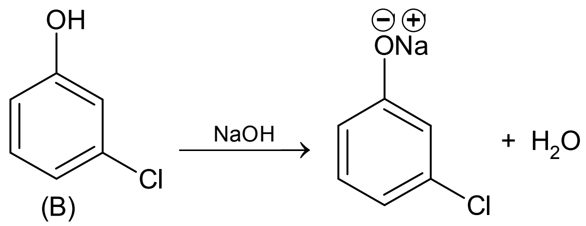
**10.** Three compounds (A), (B) and (C) are as follow :



Compound (A) reacts with $NaHCO_3$ to liberate $CO_2$ gas. Compound (B) reacts with NaOH. Identify the remaining compound.

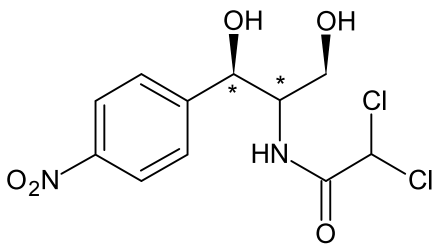
**Sol.**





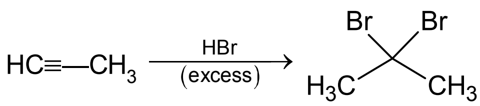
**11.** Number of chiral centres present in chloramphenicol ?

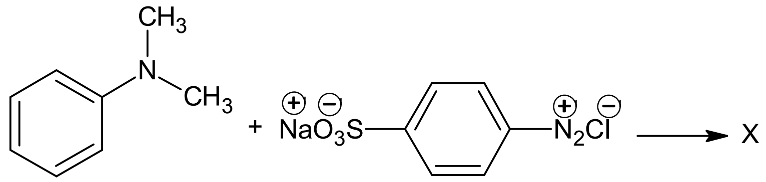
**Sol.** **2**



**12.** Methyl acetylene $\xrightarrow[\text{(excess)}]{\text{HBr}}$ Find the product

**Sol.**



**13.** 

Find the correct statement regarding 'X' :

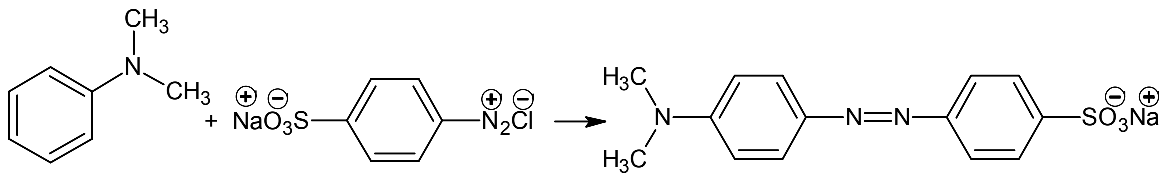
(a) Used as indicator

(b) To distinguish phenol

(c) to distinguish carbohydrate

(d) Use as food colour

**Sol.** Used as indicator



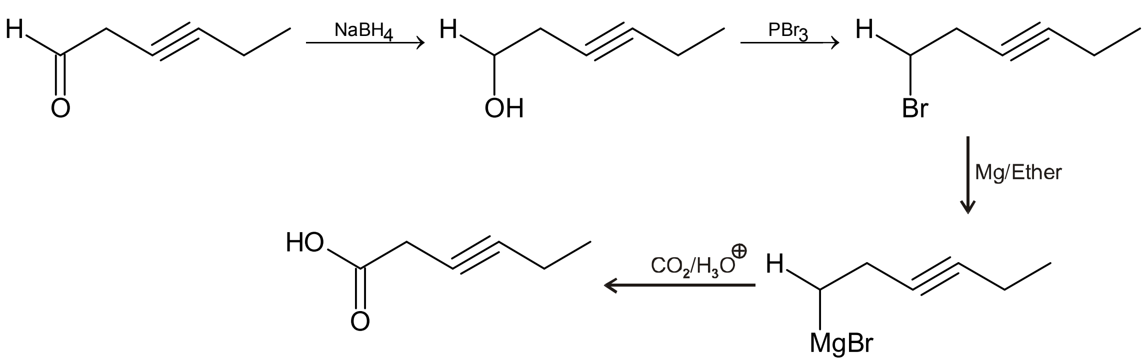
**14. Match the correct option :**

| **Vitamins** | **Disease** |
|---|---|
| (a) Riboflavin | (p) Beri-Beri |
| (b) Ascorbic acid | (q) Rickets |
| (c) Thiamine | (r) Scurvy |
| (d) Calciferol | (s) Cheilosis |

**Sol.** (a)→(s), (b)→(r), (c)→(p), (d)→(q)

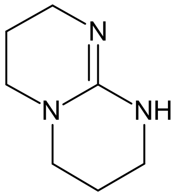
| | |
|---|---|
| (a) Riboflavin | (s) Cheilosis |
| (b) Ascorbic acid | (r) Scurvy |
| (c) Thiamine | (p) Beri-Beri |
| (d) Calciferol | (q) Rickets |

**15.** $$\text{Hex-3-ynal} \xrightarrow[\text{3. Mg/Ether} \quad \text{4. } CO_2/H_3O^{\oplus}]{\text{1. } NaBH_4 \quad \text{2. } PBr_3} \text{Product}$$

**Sol.** 

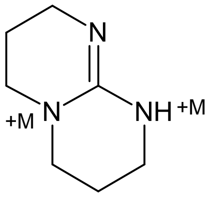
**16.** Find decreasing order of $pK_b$ for following :

(a) $CH_3–NH–CH_3$

(b) 

(c) $NH=CH—NH_2$

**Sol.** (a) $\overset{+I}{C}H_3–NH–\overset{+I}{C}H_3$

(b) 

(c) $NH=CH–\overset{+M}{N}H_2$

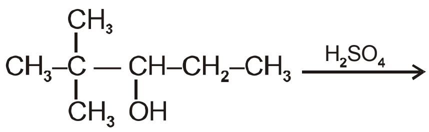
Order of $K_b$ : (b) > (c) > (a)

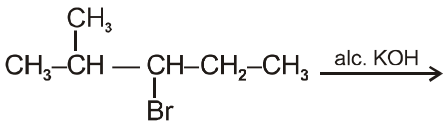
Basic strength $\propto$ +M / + H / +I

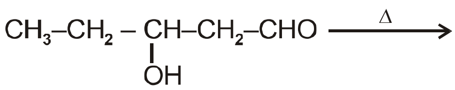
$$pK_b \propto \frac{1}{K_b}$$

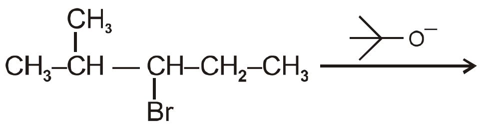
Order of $pK_b$ : (a) > (c) > (b)

**17.** In which of the following satzeff alkene is **not** major product obtained :

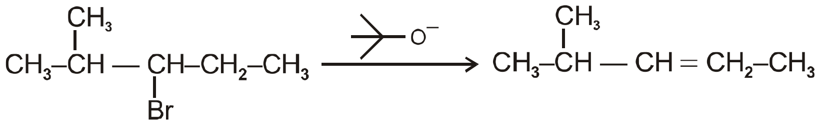
(a) 

(b) 

(c) 

(d) 

**Sol.** only (d)



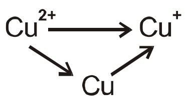
**18.** Vapour pressure of pure $CS_2$ and acetone are 345 mmHg and 512 mmHg respectively. Their mixture have observed vapour pressure 600 mm Hg. find incorrect option.

**Sol.** It is a mixture with +ve deviation from roult's law

**Incorrect option** $\Rightarrow$ on mixing 100 ml of each liquid $V_{mix}$ < 200 ml

**19.** If $E^o_{Cu^{+2}/Cu} = 0.34V$ & $E^o_{Cu^{+}/Cu} = 0.522V$ then calculate E° for $Cu^{+2} + e^- \longrightarrow Cu^+$

**Ans.** **0.158 V**



$$E^o_{Cu^{+2}/Cu^{+}} = 2 \times 0.34 - 0.522 = 0.158$$

**20.** Find number of orbitals corresponding to n = 5, $m_s = +\frac{1}{2}$

**Ans.** **25**

**21.** 1 μg of radioactive Sr was injected in new born baby. Find time in which 90% of Sr decomposes (Given $t_{1/2}$ = 6.93 yrs.)

**Ans.** **23.03 yr**

**Sol.** $\frac{0.693}{6.93} \times t = \ln\frac{100}{10}$ = 2.303 × 10 = 23.03 yr.

**22.** One question related to incorrect statement based on Dalton's atomic theory.

**23.** 9.8 gm $H_2SO_4$ was dissolved in 100 L water to form solution A and 4 gm NaOH was dissolved in another 100 L water to form solution B. If 10 L of solution A is mixed with 40 L of solution B. Find pH of final solution.

**Ans.** **10.6**

**Sol.**

| | | | | |
|---|---|---|---|---|
| $H_2SO_4$ | + | 2 NaOH | $\longrightarrow$ | $Na_2SO_4 + H_2O$ |
| $10^{-3} \times 10$ **(LR)** | | $10^{-3} \times 40$ | | |
| – | | $20 \times 10^{-3}$ | | $10^{-3}$ |

$[OH^-] = \frac{20 \times 10^{-3}}{10 + 40} = 4 \times 10^{-4}$

pOH = 3.4

pH = 14–3.4 = 10.6

**24.** For the given reaction $A(\ell) \rightleftharpoons 2B(g)$

$\Delta U$ = 2.1 kCal mol$^{-1}$ $\Delta S$ = 20 Cal K$^{-1}$ mol$^{-1}$

Calculate $\Delta G$ at 300 K

**Ans.** **–2.7** kCal mol$^{-1}$

**Sol.** $\Delta H = \Delta U + \Delta n_g RT = 2.1 + 2 \times 2 \times \frac{300}{1000}$ = 3.3 kcal mol$^{-1}$

$\Delta G = \Delta H - T\Delta S = 3.3 - \frac{300 \times 20}{1000} = -2.7$ kCal/mol

# Memory Based_JEE Main Online Test_07-01-20_Morning

## MATHEMATICS

**1.** No. of 6 digit numbers can be formed using all the digits 1, 3, 5, 7, 9 at a time is :

**Ans.** $\frac{5}{2}\cdot 6!$

**Sol.** ${}^5C_1 \times \frac{6!}{2!} = \frac{5}{2}6!$

**2.** If $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{1/3}=0$ and $x^k + y^k = a^k$, then find value of 'k'

**Ans.** $\frac{2}{3}$

**Sol.** $\int y^{-1/3}dy = -\int x^{-1/3}\,dx$

$$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = \frac{-x^{\frac{2}{3}}}{\frac{2}{3}} + C \qquad y(0) = a$$

$$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = \frac{-x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{a^{\frac{2}{3}}}{\frac{2}{3}}$$

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

**3.** If '$\alpha$' is root of $x^2 + x + 1 = 0$ and $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$ then $\mathbf{A^{31}}$ is equal to -

**Ans.** $\mathbf{A^3}$

**Sol.** $A^2 = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

therefore $A^{31} = A^{28}.\ A^3 = (A^4)^7\ .\ A^3 = A^3$

**4.** Variance of first n natural numbersis 10 and variance of first m even natural nos is 16 then find m + n.

**Ans** 18

**Sol.** $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$10 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$

$10 = \frac{2n^2+3n+1}{6} - \frac{(n^2+1+2n)}{4}$

$120 = 4n^2 + 6n + 2 - 3n^2 - 3 - 6n$

$n^2 = 121$

$n = 11$

Also $16 = \frac{4\left(\frac{m(m+1)(2m+1)}{6}\right)}{m} - \left(\frac{2m(m+1)}{2m}\right)^2$

$16 = \frac{2}{3}(2m^2 + 3m + 1) - (m^2 + 1 + 2m)$

$48 = 4m^2 + 6m + 2 - 3m^2 - 3 - 6m$

$48 = m^2 - 1$

$m^2 = 49$

$m = 7$

$\therefore$ m + n = 18

**5.** If system of equations :

x + ay + 2az = 0

x + by + 3bz = 0

x + cy + 4cz = 0

has non-trivial solution then

(1) a, b, c are in A.P.

(2) a, b, c are in G.P.

(*3) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(4) a + b + c = 0

**Sol.** $\begin{vmatrix} 1 & a & 2a \\ 1 & b & 3b \\ 1 & c & 4c \end{vmatrix} = 0$

$\begin{vmatrix} 0 & a-b & 2a-3b \\ 0 & b-c & 3b-4c \\ 1 & c & 4c \end{vmatrix} = 0$

$(3b - 4c)(a - b) - (b - c)(2a - 3b) = 0$

$(3ab - 3b^2 - 4ac + 4bc) - (2ab - 3b^2 - 2ac + 3bc) = 0$

$ab - 2ac + bc = 0$

$ab + bc = 2ac \Rightarrow b = \frac{2ac}{a+c}$

**6.** In an Ellipse, distance between both the focus is 6 and distance between directrix is 12 then find length of latus rectum.

**Ans.** $3\sqrt{2}$

**Sol.** $2ae = 6 \Rightarrow ae = 3$ ......(1)

$\frac{2a}{e} = 12 \Rightarrow \frac{a}{e} = 6$ . ......(2)

from (1) and (2) $a^2 = 18$, $b^2 = 9$

$$\text{L.R.} = \frac{2b^2}{a} = \frac{2.9}{3\sqrt{2}} = 3\sqrt{2}$$

**7.** $49^k + 1$ is a factor of $1 + 49 + 49^2 + . ...... + 49^{125}$ then max value of k is

**Ans.** 63

**Sol.** $\therefore$ $1 + 49 + 49^2. .........+49^{125}$

$$= \frac{(49^{126} - 1)}{49 - 1} = \frac{(49^{63} - 1)(49^{63} + 1)}{48}$$

$k_{max} = 63$

**8.** If $\left(\frac{dy}{dx} - 1\right) e^y = e^x$ and $y(0) = 0$ then find $y(1)$

**Ans.** $\log 2 + 1$

**Sol.** $\frac{dy}{dx} = e^{x-y} + 1$

$x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

$1 - \frac{dt}{dx} = e^t + 1$

$\frac{dt}{dx} = -e^t$

$\int e^{-t} dt = -\int dx$

$e^{-t} = x + C$

$e^{-x+y} = x + C$

Put $x = 0$, $y = 0$ $c = 1$

$e^{-x+y} = x + 1$

$e^{-1+y} = 2$

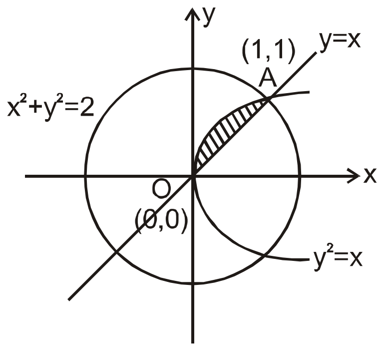
$y - 1 = \log 2 \Rightarrow y = \log 2 + 1$

**9.** Find area of circle $x^2 + y^2 = 2$ which do not lie between the curves $y^2 = x$ and $y = x$

**Ans.** $\frac{(12\pi - 1)}{6}$ (Square units)

**Sol.** Required area = Area of circle – Area bounded by given two curves

$$= 2\pi - \left[\int_0^1 (\sqrt{x} - x)\,dx\right]$$




$$= 2\pi - \left[\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1$$

$$= 2\pi - \left[\frac{2}{3} - \frac{1}{2}\right]$$

$$= 2\pi - \frac{1}{6}$$

$$= \frac{(12\pi - 1)}{6}$$

**10.** Common tangent on $y^2 = 4x$ & $x^2 = 2by$ is $y = mx + 4$. Then value of b is

**Ans.** –128

**Sol.** Tangent on $y^2 = 4x$ is $y = mx + \frac{1}{m}$

$\therefore$ given tangent is $y = mx + 4$ .....(1)

$\therefore$ $\boxed{m = \frac{1}{4}}$

Tangent on $x^2 = 2by$ is

$$y = mx - \frac{bm^2}{2}$$

Put $m = \frac{1}{4}$

$$y = \frac{x}{4} - \frac{b}{32}$$

Compare with (1)

$$\frac{-b}{32} = 4$$

$b = -128$.

**11.** If F(a + b + 1 – x) = F(x) then

$$\frac{1}{a+b}\int_a^b x(F(x)+F(x+1))dx$$ is equal to

(1) $\int_{a+1}^{b+1} F(x)dx$ (*2) $\int_{a-1}^{b-1} F(x+1)dx$ (3) $\int_{a+1}^{b+1} F(x+1)dx$ (4) $\int_{a-1}^{b-1} F(x)dx$

**Ans.** (2)

**Sol.** $I = \frac{1}{a+b}\int_a^b x.\left(F(x)+F(x+1)\right)dx$ ....(1)

$I = \frac{1}{a+b}\int_a^b (a+b-x)\left(F(a+b-x)+F(a+b-x+1)\right)dx$

(1) + (2)

$2I = \int_a^b \left(F(x)+F(x+1)\right)dx$

$2I = \int_a^b F(x)dx + \int_a^b F(x+1).dx$

$2I = \int_a^b F(x)dx + \int_a^b F(a+b-x+1).dx$

$2I = \int_a^b F(x)dx + \int_a^b F((x)dx$

$I = \int_a^b F(x)dx$

x = t + 1

$I = \int_{a-1}^{b-1} F(t+1)dt$

**12.** $f:[-7,0]\to R$ is differentiable between and $f'(x)\le 2$. If $f(7)=-3$ then find range of $f(-1)+f(0)$

**Ans.** **$(-\infty, 20]$**

**Sol.** $\dfrac{f(-1)-f(-7)}{-1-(-7)}=f'(c_1)\le 2$

$f(-1)-f(-7)\le 12$

$f(-1)\le 9$ .......(1)

$\dfrac{f(0)-f(-7)}{0-(-7)}=f'(c_2)\le 2$

$f(0)-f(-7)\le 14$

$f(0)\le 11$ .........(2)

from (1) and (2)

$f(0)+f(-1)\le 20$

**13.** $(p\to q)\wedge(p\to \sim q)$ is equivalent to :

**Ans.** **~p**

**Sol.**

| p | q | ~q | $p\to q$ | $p\to\sim q$ | $(p\to q)\wedge(p\to\sim q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| F | T | F | T | T | T |
| F | F | T | T | T | T |

**14.** If $\alpha, \beta$ are roots of the equation $(k+1)\tan^2x-\sqrt{2}\lambda\tan x=-k+1$ and $\tan^2(\alpha+\beta)=50$. Find '$\lambda$'

**Ans.** **10**

**Sol.** $(k+1)\tan^2x-\sqrt{2}\lambda\tan x+k-1=0$ $\big\langle\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

$\tan\alpha+\tan\beta=\dfrac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha.\tan\beta=\dfrac{k-1}{k+1}$

$\tan^2(\alpha+\beta)=50$

$\left(\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}\right)^2=50$ ; $\left(\dfrac{\frac{\sqrt{2}\lambda}{k+1}}{2-\frac{k-1}{k+1}}\right)^2=50$ ; $\left(\dfrac{\sqrt{2}\lambda}{2}\right)^2=50$

$\lambda^2=100$

$\lambda=10$

**15.** If $g(x) = x^2 + x - 1$ and $g(F(x)) = 4x^2 - 10x + 5$ then $F(5/4)$ is equal to

**Ans.** $-1/2$

**Sol.** $g(F(x)) = 4x^2 - 10x + 5$

$g(F(5/4)) = 4(5/4)^2 - 10(5/4) + 5$

$g(F(5/4)) = \frac{25}{4} - \frac{25}{2} + 5$

$g(F(S/4)) = 30 - \frac{25}{4} - \frac{35}{2} - \frac{5}{4}$

let $F(5/4) = t$

$g(t) = -\frac{5}{4}$

$4t^2 + 4t - 4 = -5$

$4t^2 + 4t + 1 = 0$

$(2t + 1)^2 = 0$

$t = -\frac{1}{2}$

$F(5/4) = -\frac{1}{2}$

**16.** $y = \sqrt{\frac{2(\tan x + \cot x)}{1 + \tan^2 x} + \frac{1}{\sin^2 x}}$, $x \in \left(\frac{3\pi}{4}, \pi\right)$

Find $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$

(*1) 4 (2) 8 (3) $-4$ (4) $-\frac{1}{4}$

**Sol.** $y = \sqrt{\frac{2(\sin^2 x + \cos^2 x)}{\sec^2 x . \sin x \cos x} + \text{cosec}^2 x}$

$= \sqrt{2 \cot x + 1 + \cot^2 x}$

$y = |1 + \cot x|$

$y = -1 - \cot x$ as $x \in \left(\frac{3\pi}{4}, \pi\right)$

$\frac{dy}{dx} = \text{cosec}^2 x$

At $x = \frac{5\pi}{6}$

$\frac{dy}{dx} = 4$

**17.** Let p be a plane passing through the points (2, 1, 0) (4, 1, 1) & (5, 0, 1). R(2, 1, 6) is any point. Then image of R about the plane p is.

**Ans.** **(4, 3, 2)**

**Sol.** $\because$ Let plane is

$a(x-4) + b(y-1) + c(2-1) = 0$ .....(1)

Put (5, 0, 1)

$a - b = 0$

$\Rightarrow \boxed{a = b}$

Put (2, 1, 0)

$-2a - c = 0$

$c = -2a$

Put in (1) we get plane p is

$(x-4) + y - 1 - 2(2-1) = 0$

$x + y - 2z - 3 = 0$

Let : Image of R about plane p is $\theta(x, y, z)$

$$\therefore \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{1+1+4}$$

$$x - 2 = y - 1 = \frac{z-6}{-2} = 4$$

$x = 6,\ y = 5,\ z = -2.$

**18.** Let $a_1, a_2, a_3, a_4, a_5$ are in A.P. and $a_1 + a_2 + a_3 + a_4 + a_5 = 25$

$a_1 a_2 a_3 a_4 a_5 = 2520$. If one the number is $-\frac{1}{2}$, then find largest number.

**Ans.** **16**

**Sol.** $(5-2d)(5-d)\,5(5+d)(5+2d) = 2520$

$(25-d^2)(25-4d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$(4d^2 - 121)(d^2 - 1) = 0$

$d = \pm\frac{11}{2}$ $\quad$ $d = \pm 1$ (Not possible)

$d = \frac{11}{2} \Rightarrow$ Largest number $= 5 + 2d = 5 + 11 = 16$

**19.** If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where z = x + iy, then the point (x, y) lies on

(1) Straight line with slope $-\frac{2}{3}$

(2) Straight line with slope $\frac{3}{2}$

(*3) Circle whose diameter is $\frac{\sqrt{5}}{2}$

(4) Circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$

**Ans.** (3)

**Sol.** $\text{Re}\left(\frac{z-1}{2z+i}\right)=\text{Re}\left[\left(\frac{(x-1)+iy}{2x+(2y+1)i}\right)\left(\frac{2x-(2y+1)i}{2x-(2y+1)i}\right)\right]$

$=\frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2}=1$

$(2x^2 - 2x) + (2y^2 + y) = 4x^2 + 4y^2 + 4y + 1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y+\frac{1}{2}=0$

$r=\sqrt{g^2+f^2-c}=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\frac{\sqrt{4+9-8}}{4}=\frac{\sqrt{5}}{4}$

**20.** In the product $(1 + x + x^2 + ........x^{2n})(1 - x + x^2 - x^3. ........+ x^{2n})$ the sum of coff. of even powers of 'x' is 61 then find 'n'.

**Ans.** **30**

Put x = 1

$2n + 1 = a_0 + a_1 + a_2 + a_3. ......$ ....(1)

Put x = –1

$2n + 1 = a_0 - a_1 + a_2 + a_3. ......$ ....(2)

Add (1) and (2)

$2(2n + 1) = 2(a_0 + a_2 + a_4. ....) \Rightarrow 2n + 1 = 61 \Rightarrow n = 30.$

**21.** Find value of $\lim\limits_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$

**Ans.** 36

**Sol.** $3^{x/2} = t$

$$\lim_{t\to 3}\frac{t^2+\frac{27}{t^2}-12}{\frac{1}{t}-\frac{3}{t^2}}=\lim_{t\to 3}\frac{t^4-12t^2+27}{t-3}$$

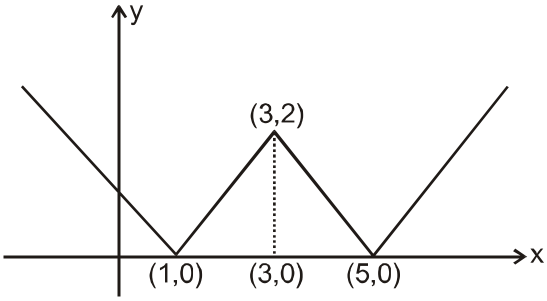
$$=\lim_{t\to 3}\frac{(t^2-9)(t^2-3)}{t-3}$$

$$=\lim_{t\to 3}(t+3)(t^2-3)=36$$

**22.** Let 'S' be the set of point where F(x) = |2 – |x – 3|| is non-differentiable then the vaelu of $\sum_{x\in S} F(F(x))$

**Ans.** 3

**Sol.** F(x) = |2 – |x – 3||




x = 1, 3, 5 (points of non differentiability)

$$\sum_{x\in S} F(F(x)) = F(F(1)) + F(F(3)) + F(F(5))$$

$$= 1 + 1 + 1$$

$$= 3$$

**23.** Let A(1, 0) B(6, 2) C$\left(\frac{3}{2}, 6\right)$ are vertices of ΔABC if 'P' is a point inside the triangle such that ΔPAB, ΔPBC, ΔPCA have equal area then length of line segment PQ where 'Q' is point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$ is

**Ans.** 5

**Sol.** 'P' is centroid of ΔABC ≡ $\left(\frac{17}{6}, \frac{8}{3}\right)$

$$PQ = \sqrt{\left(\frac{17}{6}+\frac{7}{6}\right)^2 + \left(\frac{8}{3}+\frac{1}{3}\right)^2}$$

$$PQ = \sqrt{16+9} = 5$$

**24.** Unbiased coin is tossed '5' times. Suppose that a variable 'x' atlends the value k, when 'k' consecutive hends are obtained for k = 3, 4, 5, otherwise 'x' take the value –1 ; then the expected value of 'x' is :

**Ans.** $\frac{1}{8}$

**Sol.** p(x = 3) = HHHTX + THHHT + XTHHH

$$p(x = 3) = \frac{1}{16} + \frac{1}{32} + \frac{1}{16} = \frac{5}{32}$$

$$p(x = 4) = HHHHT + THHHH = \frac{1}{32} + \frac{1}{32} = \frac{1}{16}$$

$$p(x = 5) = HHHHH = \frac{1}{32}$$

$p_1 + p_2 + p_3 + \frac{5}{32} + \frac{1}{16} + \frac{1}{32} = 1$

$\left(p_1 + p_2 + p_3 = \frac{3}{4}\right)$

$E(x) = \sum x_i\, p(x_i) = (-1)p_1 + (-1)p_2 + (-1)p_3 + \frac{15}{32} + \frac{4}{16} + \frac{5}{32}$

$E(x) = \frac{1}{8}$